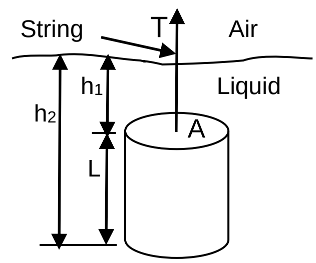
Objective:
The objective of this lab is to experimentally support Archimedes Principle of buoyancy and to use it to determine the densities of different materials.

Theory:
In a gravitational field the pressure exerted by a fluid depends on the gravitational acceleration, the density or the fluid, and the depth in the fluid. If an extended object is submerged in a fluid, the horizontal force components on the object cancel out, but the vertical force components generally do not, causing a net upward force called the buoyant force (see derivation below).

The magnitude of the buoyant force is equal to the weight of the fluid which is displaced by the object. This can be understood as follows: Consider the fluid (water in this experiment) which will be displaced by the object. It (the water) is in equilibrium. That means that the buoyant force exerted on it must be equal to the weight of the water displaced. Anything which replaces that volume of water will experience the same buoyant force that the missing water would experience. The proof:

Consider a solid cylinder of length L, end surface area A, and of density ρ_s which is suspended from a string and totally submerged in a liquid of density ρ as shown to the right.



A free-body analysis of the cylinder is in order. Pascal's Law states that the pressure of a fluid at depth h is given by:

$$P = \rho gh.$$

On the top of the cylinder, the force is $F = PA$, or $F_t = \rho gh_1A$, and on the bottom it's $F_b = \rho gh_2A$. When we add up all of the vertical forces on the cylinder, in the upward direction we have the tension in the string, T, and the force of the fluid on the bottom, F_b. In the downward direction, we have the weight of the cylinder, mg where m is the cylinder's mass and the fluid's force on the top, F_t . Since the cylinder is in equilibrium, we can write:

$$\Sigma F_y = T + F_b - mg - F_t = 0 \quad \text{or} \quad T + \rho gh_2A - mg - \rho gh_1A = 0 \quad \text{or} \quad T = mg - \rho gA(h_2 - h_1)$$

But, $h_2 - h_1 = L$, the length of the cylinder, and AL is the volume of the cylinder, V. This gives us:

$$T = mg - \rho gV.$$

But ρV is the mass of a volume of fluid which is the same volume as that which is displaced by the cylinder. And when that mass is multiplied by g, it becomes the weight of the fluid displaced by the cylinder. The difference between the cylinder's true weight (mg) and it's apparent "weight" when immersed in a fluid (T) is called the buoyant force. So, *the buoyant force is equal to the weight of the fluid displaced*. This is a statement of Archimedes Principle. We derived this for a cylinder, but it's true for any object of any shape.

If we divide both sides of the equation $T = mg - \rho gV$ by g, T/g becomes the mass you will measure with the object immersed in the water. We will call this the "wet mass" and label it m_w. And, we will call m_s the "dry mass." Therefore, we can write the above equation in the following words: "The mass of the water displaced (ρV) equals the dry mass minus the wet mass." The goal of the first part of this experiment is to validate this statement.

Using this definition of the "wet mass", we write: $m_w = T/g$ and thus we rewrite the equation above as:

$$m_w = m_s - \rho V \quad \text{or} \quad \rho V = m_s - m_w \qquad \textbf{(Eq. 1)}$$

which is an equation for the quoted statement above. Finally, we recall that the mass of the cylinder, m_s, can be written as $m_s = \rho_s V$ where V is the volume of the cylinder and ρ_s is the density of the cylinder's material. Solving this for V, we have $V = m_s / \rho_s$ which we substitute into the above equation to get:

$$m_w = m_s - \rho(m_s / \rho_s)$$ If we solve this equation for ρ_s we get:

$$\rho_s = \rho m_s / (m_s - m_w) \qquad \textbf{(Eq. 2)}$$

We can use (Eq. 2) to solve for the densities of objects. We will assume that the density of water, ρ, is 1000 kg/m^3.

Equipment List:

1 liter plastic beaker
Electronic balance (one per team)
Graduated cylinder
Small Beaker
Support stand
Four objects to be tested (they will be on a sheet with numbers to identify each)
String to suspend the objects

Procedure:

1. Prior to the experiment, the lab instructor will have filled your 1 liter plastic beaker with water. Also, prior to the experiment, the electronic balance will have had a hook screwed into the bottom of the unit, or the balance has a small, orange plastic hook on the bottom. Turn the balance over to find this hook. The balance will be sitting on a ring attached to a ring-stand. You may raise or lower the ring to fully immerse the object. Please be careful not to allow the balance to fall to the table. It is very easily damaged.
2. Zero the balance and record the mass of your dry graduated cylinder.
3. Measure and record the "dry" mass of each of your four objects by placing each on top of the electronic balance.
4. Use your small beaker to dip enough water to approximately half-fill the graduated cylinder (or find a restroom sink). Do your best to adjust the water level in the cylinder to read exactly 60 ml.
5. With a string tied to Object 1, lower it completely into the cylinder and measure and record the new water level (in milliliters).
6. Pour some of the cylinder's water back into the beaker, leaving in the cylinder the exact amount of water that Object 1 displaced. You may need to use the small beaker to accomplish this. Then place the graduated cylinder back on the scale and measure and record its new mass with the added water. The additional mass you measure should be the mass of the water in the cylinder (see Calculation 2).
7. Suspend each of your four objects from the string hanging beneath the balance, fully immersing each in the 1 liter beaker. Don't allow the objects to touch the sides or the bottom of the beaker. Also, note carefully that some of the objects have cavities drilled in them. These cavities can trap air and cause serious errors. Make certain that you suspend these objects horizontally so that the water will fill the cavity. Measure and record the "wet" mass of each object while immersed. You may need to make a lasso loop with the string to hold each object.

Calculations:

1. From Procedures 2, 5, and 6, subtract the mass of the dry graduated cylinder from its mass with the displacement water in it to get the mass of the displaced water. Find the percent difference between the calculated mass of the displaced water and the "lost" mass ($m_s - m_w$) for Object 1 in Procedure 7.
2. Calculate the mass "lost" for each object while immersed in the water. This will be the difference between each object's "dry" mass and its "wet" mass as determined in Procedure 7.

3. Using Eq 2., calculate the density of each of your four objects.
4. Compare the calculated densities with those of known materials, using a table in your text or other reference, and identify the material of each of your four objects by Object number. There is no need to calculate a percent error since there are so many alloys of these materials, each having a different density.

Questions:

1. Referencing Eq. 1, does the mass difference measured for Object 1 in Procedure 7 equal the mass of the water it displaced as calculated in Procedure 6. Why or why not? Explain any difference.

2. Looking at the derivation above, what happens if ρ_s is less than ρ?